A discrete math course with early graph theory

Oscar Levin

University of Northern Colorado

Discrete Mathematics in the Undergraduate Curriculum
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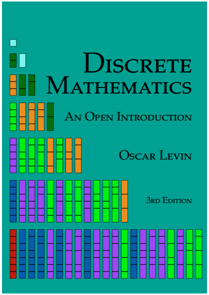
Not a shameless plug

Discrete Mathematics: An Open Introduction



A free and open source introductory discrete textbook with interactive online ebook, pdf ebook, and print editions.

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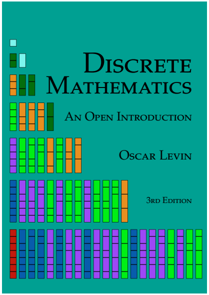

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Here is why you might <u>not</u> want to use this book.

A traditional schedule

1. Preliminaries (statements, sets, functions)
2. Counting
3. Sequences (also induction)
4. Logic and proof techniques
5. Graph theory

Some traditional drawbacks

- Preliminaries are boring.
- Counting is hard.
- Proofs about number theory are unnecessary.
- We run out of time for the good stuff.

A better way?

5 Graph theory
1 Preliminaries (statements, sets, functions)
2 Counting
3 Sequences (also induction)
4 Logic and proof techniques

What about prerequisites?

There is a reason graph theory usually goes last.

You need proof by induction and proof by contradiction.

So probably also other proof techniques and logic.

Some set theory (at least notation) is helpful. Maybe functions?

The solution: do all of these as you go, motivated by the need to do graph theory.

Starting to get messy

5 Graph theory
1 Preliminaries (statements, sets, functions)
4 Logic and proof techniques (also induction)
2 Counting
3 Sequences (also induction, again)

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5 Graph theory

1 Preliminaries (statements, sets, functions)

4 Logic and proof techniques (also induction)

2 Counting

3 Sequences (also induction, again)

Example 1: Who cares about truth tables?

Consider the statement, "If a graph is regular or 3-connected, then it is not planar." You do not need to know what these terms mean to complete this problem.

(a) Make a truth table for the statement $(R \vee C) \to \neg P$.

(b) If you believed the statement was *false*, what properties would a counterexample need to possess? Explain by referencing your truth table.

(c) If the statement were true, what could you conclude about the graph W_7, which is definitely planar? Again, explain using the truth table.

Example 2: Proofs *do* grow on trees

If there is at most one path between any pair of vertices, then the graph is a forest. Write the first and last line of a proof of this statement provided you were to prove it *directly*, *by contrapositive* or *by contradiction*. Then give a proof using an appropriate style.

Repeat the above for proving the converse: **if the graph is a forest, then there is at most one path between any pair of vertices.**

Prove that any tree with at least two vertices has at least one leaf (i.e., a degree 1 vertex). Note this can be phrased as an implication: **if G is a tree with at least two vertices, then G has at least one vertex of degree 1.**

Example $k+1$: Understanding induction

Prove the any tree with v vertices has $v-1$ edges.

Prove that every tree is bipartite.

Prove Euler's formula for planar graphs.

What about counting?

The chromatic polynomial: how many k-colorings are there of a given labeled graph?

$$P(K_n, k) = P(n, k) \quad P(P_n, k) = k(k-1)^n \quad P(C_n, k) = ??$$

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Later: you can do Principle of Inclusion/Exclusion and the Deletion-Contraction recursion.

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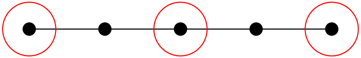

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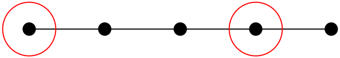

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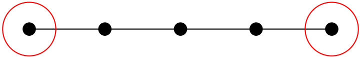

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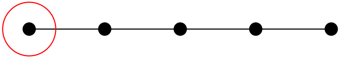

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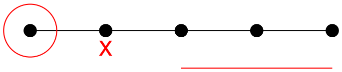

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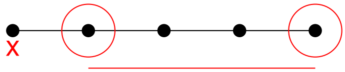

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The final exam average was 5% points higher than the previous semester.

Student comments were generally positive, but pointed out that it was confusing to jump around.

A compromise

Spend one week introducing the *problems and language* of graph theory, without doing any of the theory.

Then go back to the traditional schedule, and simply sprinkle graphs on everything.

I don't know what a textbook with this design would look like.

Also: be careful to not miss out on the graph theory *content*.

The End

Thanks!

Slides (and a textbook):

discrete.openmathbooks.org/talks.php